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reprinted from
nature

Reprinted from Nature, Vol. 304, No. 5925, pp. 434-436, 4 August 1983

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Recently reported analyses of $5\frac{1}{2}$ yr of orbital data for the laser geodynamics satellite (Lageos) have revealed a residual acceleration in the node of its orbit in the amount -14.7 ± 1.3 m arc s yr^{-2}. This acceleration is predominantly attributable to a secular decrease of the degree two component of the Earth's gravitational potential field J_2. The implied rate of change of J_2 is $(-3.5\pm0.3)\times10^{-11}$ yr^{-1}. The following analysis argues that this effect is due to Pleistocene deglaciation and that it can be used to constrain strongly the viscosity of the Earth's deep mantle.

The question as to the value of the Earth's viscosity beneath a depth of 670 km remains an important issue. This depth is marked by the presence of a solid–solid phase transition from the low-pressure phase spinel to the high-pressure mixture of the minerals perovskite and magnesiowustite[1]. Because this phase transition probably has negative Clapeyron slope and because such transitions appear not to provide any significant impediment to thermal convection through them[2], it is expected that the mantle convection currents responsible for continental drift and seafloor spreading will be whole mantle in scale.

One physical effect which could undermine this argument in favour of whole mantle convection is the influence of viscosity stratification. If the viscosity of the lower mantle were substantially higher than that of the upper mantle, the circulation could be effectively confined to the upper region. That this situation may in fact exist is a view which has been quite firmly held by some authors.

The purpose here is to provide a geophysical interpretation of a new observational datum which, when combined with previously reported results, seems to rule out the possibility that the viscosity of the lower mantle is significantly higher than that of the upper mantle. This new datum concerns a particular property of the orbit of the artificial Earth satellite Lageos which was launched in 1976 into a nearly circular and drag-free orbit about 1 Earth radius above the surface. Very recently two different analyses have appeared[3,4] which establish the presence in the time series for the location of the node of the orbit (Ω), a residual secular acceleration after the data have been corrected for known effects.

Of the recently published analyses of $\ddot{\Omega}$, that provided by Yoder *et al.*[4] is probably the more accurate because it was based on long arcs of tracking data. Two different estimates of $\ddot{\Omega}$ have been obtained based on time scales UT1 derived from both Bureau Internationale de l'Heure (BIH) data and from Lunar Laser Ranging (LLR) observations. The reported accelerations are[4]:

$$\ddot{\Omega}(\text{LLR}) = -14.7 \pm 1.3 \quad \text{m arc s yr}^{-2} \tag{1a}$$

$$\ddot{\Omega}(\text{BIH}) = -18.2 \pm 1.6 \quad \text{m arc s yr}^{-2} \tag{1b}$$

As it is generally acknowledged that the UT1 based on LLR data is superior to that derived from BIH data, we shall accept equation (1*a*) as the best current estimate of the residual acceleration. On the basis of the evolution equation satisfied by the keplerian element Ω[5], it may be shown that a non-zero value of $\ddot{\Omega}$ implies a time variation of the zonal coefficients J_2, J_4, J_6, and so on in the spherical harmonic expansion of the Earth's gravitational potential field. The dominant contribution is from $\dot{J}_2$ which has been inferred to have the value[4]:

$$\dot{J}_2(\text{LLR}) = (-3.5 \pm 0.3) \times 10^{-11} \quad \text{yr}^{-1} \tag{2a}$$

$$\dot{J}_2(\text{BIH}) = (-4.4 \pm 0.4) \times 10^{-11} \quad \text{yr}^{-1} \tag{2b}$$

As a change in J_2 implies a change of the polar moment of inertia C such that $\delta J_2 = (3/2)\ \delta C/MR^2$, with M and R the Earth's mass and radius respectively, and since the angular momentum $C\omega$ must be conserved in the absence of external torques, the change in J_2 is associated with a change in the Earth's rotation rate ω in the amount $\delta\omega/\omega = -\delta J_2 2MR^2/3C$. Although the dominant contribution to the long time scale change of the Earth's rotation rate is the deceleration due to lunar tidal dissipation in the oceans, it has been possible to extract from historical records of ancient eclipses a nontidal (NT) component in the amount[6]:

$$\left(\frac{\dot{\omega}}{\omega}\right)_{\text{NT}} = (6.9 \pm 2.6) \times 10^{-11} \quad \text{yr}^{-1} \tag{3}$$

This may be compared with the acceleration expected on the basis of the Lageos observation of $\dot{J}_2$ using the above simple relation between $\delta\omega/\omega$ and δJ_2. The Lageos data then imply

$$\left(\frac{\dot{\omega}}{\omega}\right)_{\text{NT}} (\text{LLR}) = (7.1 \pm 0.6) \times 10^{-11} \quad \text{yr}^{-1} \tag{4a}$$

$$\left(\frac{\dot{\omega}}{\omega}\right)_{\text{NT}} (\text{BIH}) = (8.8 \pm 0.8) \times 10^{-11} \quad \text{yr}^{-1} \tag{4b}$$

It is therefore clear that the newly observed $\dot{J}_2$, based on laser ranging to Lageos, is fully compatible with the nontidal acceleration obtained previously from ancient eclipse observations.

In 1966, Dicke[7] suggested that the nontidal component of the acceleration of rotation, which seemed to be required to fit the historical eclipse record, was an effect due to Pleistocene deglaciation. The last deglaciation event of the current ice age began ~18,000 yr ago. The complete disintegration of the Laurentian (Canadian) and Fennoscandian ice sheets and the partial collapse of those on Greenland and Antarctica led to a global rise in sea level of ~100 m and to a redistribution of matter in the mantle by viscous flow forced by the associated gravitational imbalance. Evidence of the isostatic adjustment (viscous flow) process is clearly revealed in the relative sea level record in the regions which were once ice-covered[8] (such as the Hudson Bay and Gulf of Bothnia). Dicke and later authors[9–11] used very simple models to invert the observed

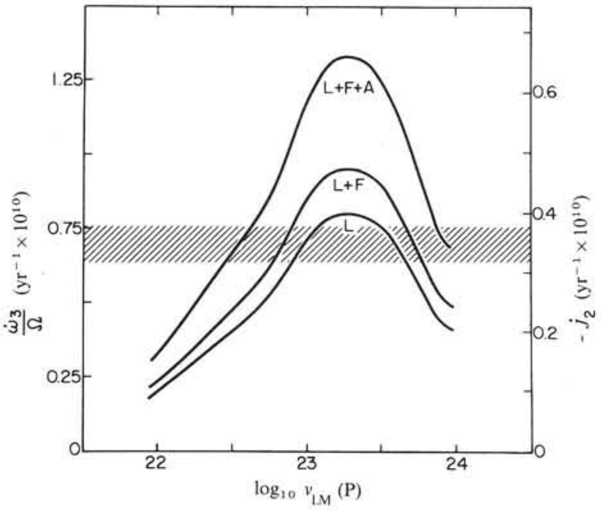

Fig. 1 The variation of the present-day predicted nontidal acceleration ($\dot{\omega}/\Omega$) and the present-day predicted $\dot{J}_2$ as a function of the viscosity of the lower mantle (ν_{LM}) for models with fixed lithospheric thickness, $L = 295.3$ km. Predictions for $L = 120$ km do not differ significantly from these. The hatched region denotes the observed values of these parameters. Calculations are shown for loading by the Laurentian ice sheet only (L), for simultaneous Laurentian and Fennoscandian loading (L+F), and including the Antarctic ice sheet also (L+F+A).

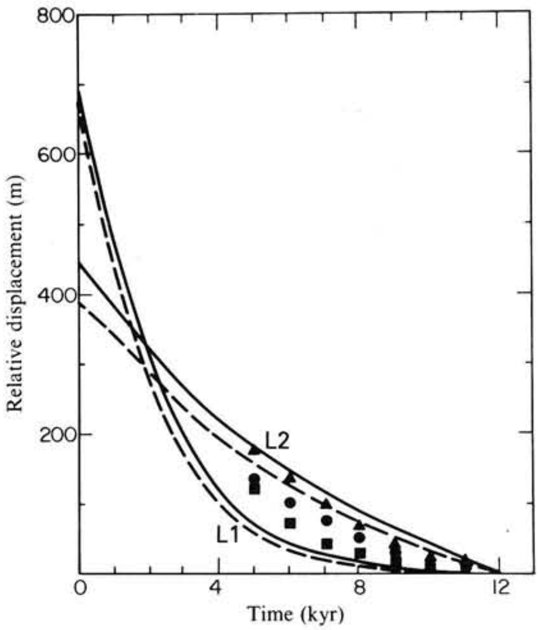

Fig. 2 Time dependence of the relative displacement at the centre of the Laurentian scale load for viscosity models L1 and L2. Solid curves are predictions based on the assumption of initial isostatic equilibrium whereas the broken curves include the effect of initial disequilibrium. The relative sea level data points obtained by radiocarbon dating of ancient beaches at Ottawa Islands (■), Churchill (●) and Castle Island (▲) respectively. These sites are all near the centre of the former Laurentide ice sheet.

nontidal acceleration to obtain an average viscosity for the entire mantle. Some of these simple calculations[9,11] suggested that it might be possible to fit the observation with either of two widely separated values of the viscosity. Here we shall summarize the results of a more sophisticated analysis of the new $\dot{J}_2$ and the old $(\dot{\omega}/\omega)_{NT}$ observations which reveals this ambiguity clearly and shows how it may be resolved.

The analysis is based on a new theory of isostatic adjustment[8] which shows that the predicted values of $(\dot{\omega}/\omega)_{NT}$ and $\dot{J}_2$ are the same to within a constant factor as

$$\dot{J}_2(t) = -\frac{4R^2}{5M}\frac{\sigma_{20}}{I_{33}^R}\left(\frac{\dot{\omega}}{\omega}\right)_{NT} \qquad (5)$$

where σ_{20} is the coefficient of second degree and zero order in the spherical harmonic expansion of the load geometry. I_{33}^R is the axial perturbation of inertia which would be associated with

Table 1 Ice sheet parameters

	Laurentia	Fennoscandia	Antarctica
Mass M_i (kg)	2.0×10^{19}	0.56×10^{19}	0.7×10^{19}
Radius α_i (deg)	15	9.5	20
Colat. α_i (deg)	30	25.5	180
Long. θ_i (deg E)	270	25.0	—

the surface load if the Earth were rigid. It is computed from the expression

$$I_{33}^R = \sum_{i=1}^{N} M_i R^2\left[\frac{2}{15}\frac{a_{20}}{a_{00}} - \frac{1}{3}\cos\alpha_1(1+\cos\alpha_1)P_2^0(\cos\theta_i')\right] \qquad (6)$$

which is obtained assuming that the ice caps are circular and discharge their melt water into ocean basins of realistic shape. M_i, α_i and θ_i' are respectively the mass, radius in degrees, and colatitude of the centroid of the ith ice cap, values of which for the main ice masses are listed in Table 1. $a_{20}/a_{00} = -0.1925$ is a ratio of spherical harmonic coefficients of the ocean function used to describe the shape of the ocean basins[6]. The detailed mathematical forms of the predictions for $\dot{J}_2$ and $(\dot{\omega}/\omega)_{NT}$ for cyclic glacial loading are available elsewhere[8,12]. Here we will restrict discussion to the results.

Figure 1 shows the theoretical predictions of present day $(\dot{\omega}/\omega)_{NT}$ and $\dot{J}_2$ as a function of the viscosity of the Earth's mantle beneath a depth of 670 km with the upper mantle viscosity held fixed at 10^{22} P. Predictions are shown for three different models denoted by L, L+F, and L+F+A which respectively contain one, two and all three ice sheets. Inspection shows that the lower mantle viscosity required by the data is either near 3×10^{22} P or near 10^{24} P. The rotation data cannot themselves be used to distinguish which of these equally plausible values is correct.

There are fortunately two different kinds of geophysical data which can be invoked to remove this ambiguity of interpretation which is intrinsic to the new $\dot{J}_2$ observation. The first of these consists of radiocarbon age-controlled relative sea level data from sites which were under or near the major ice sheets. Figure 2 compares predictions of relative sea level at the centre of a circular disk model of the Laurentian ice sheet with actual relative sea level data from three sites in and around Hudson Bay. Theoretical predictions are shown for models L1 and L2 which have lower mantle viscosities of 10^{22} P and 10^{23} P respectively and upper mantle values of 10^{22} P. The solid curves are based on the assumption of isostatic equilibrium while the dashed curves include the influence of a load history constrained by $^{18}O/^{16}O$ data from deep sea sedimentary cores[8]. Since all

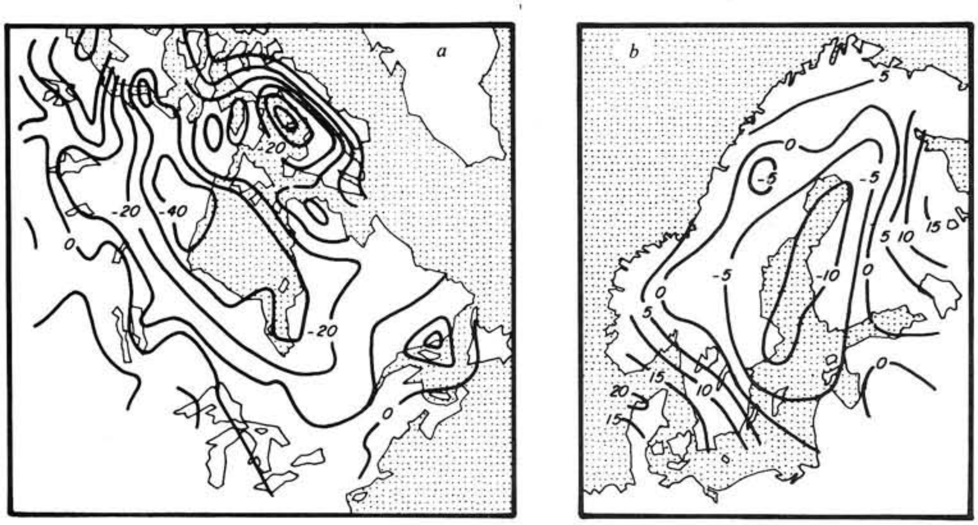

Fig. 3 Free air gravity anomalies (in mGal) over: *a*, Hudson Bay; and *b*, the Gulf of Bothnia.

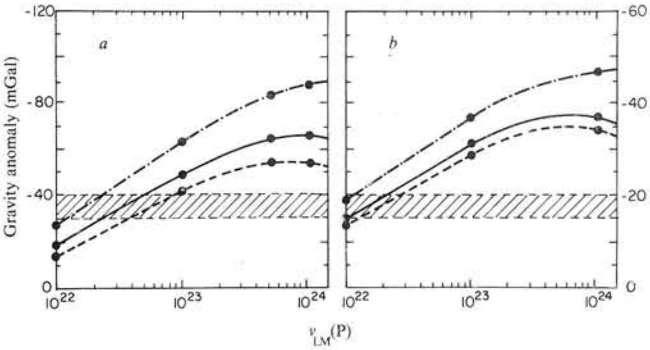

Fig. 4 Predicted present-day peak free air gravity anomaly at the centre of disk model Laurentian (*a*) and Fennoscandian (*b*) loads as a function of the viscosity of the mantle beneath 670 km depth. The viscosity of the upper mantle is held fixed at 10^{22} P. The hatched region on each plate denotes the observed peak anomaly in the corresponding region.

the data are bracketed by the predictions of L1 and L2 the implication is that lower mantle viscosity ν_{LM} is such that $10^{22}\,P \leq \nu_{LM} \leq 10^{23}$ P. These sea level data therefore exclude the higher of the two values of ν_{LM} allowed by the $\dot{J}_2$ observation and from Fig. 1 we have

$$2.7\times10^{22} \quad P \leq \nu_{LM}\,(\text{Lageos}) \leq 4.4\times10^{22} \quad P \qquad (7)$$

One further piece of observational evidence which may be used to confirm equation (7) consists of surface free air gravity anomalies over the main centres of Pleistocene deglaciation. Figure 3 shows free air gravity maps for Hudson Bay and the Gulf of Bothnia. The peak free air anomaly over the former region is near −40 mGal whereas that over the latter, appropriately corrected for large-scale bias, is near −17 mGal (ref. 8). These observed peak anomalies are shown as the cross-hatched regions on Fig. 4 where they are compared with theoretical predictions of models which differ from one another only in their values of ν_{LM}. Focusing attention on the solid lines on each plate, which are the most accurate predictions[12,13], we note that the value of the lower mantle viscosity required by the free air gravity data is in accord with that inferred from the new $\dot{J}_2$ observation.

The fact that the ν_{LM} inferred from Lageos tracking data agrees with that required by local gravity anomalies suggests that the viscosity of the lower mantle may not have appreciable large-scale heterogeneity. This is expected if this region of the Earth is well mixed by convection. Also, the observed increase of the viscosity across the seismic discontinuity at 670 km depth is most easily understood if the temperature change at this depth is adiabatic[14]. This situation will hold only if the circulation is whole mantle in scale and mass transport through the 670-km phase boundary is unimpeded.

Unfortunately, this argument for whole mantle convection is not definitive. We are not yet able to exclude the possibility of a layered convective circulation with a sharp thermal boundary layer at 670 km depth. In this scenario, the rapid increase of temperature through the boundary layer would have to be accompanied by a rather large increase of creep activation energy[14] in order that the viscosity of the lower mantle remain near the viscosity of the upper mantle as required by the new $\dot{J}_2$ observation. This model has the additional interesting feature that there would exist a thin layer of very low viscosity at the base of the upper mantle on the spinel side of the 670-km phase transformation. Work is in progress to determine whether the presence of such a layer can be ruled out on the basis of the relative sea level, free air gravity, and rotation data of postglacial rebound. The new $\dot{J}_2$ observation from Lageos laser ranging data is an important addition to the data set which we are using in a study of the inverse problem for the viscosity of the Earth's mantle.

Received 25 February; accepted 13 June 1983.

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